Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Wednesday 12 June 2024

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WME03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$, and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1.

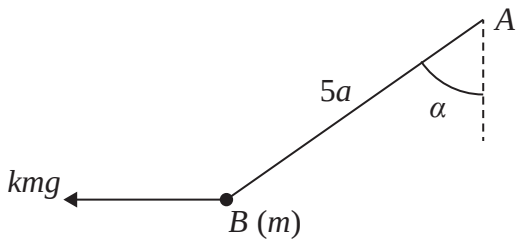

Figure 1

A light elastic string AB has natural length $4a$ and modulus of elasticity λ.
The end A is attached to a fixed point and the end B is attached to a particle of mass m.
The particle is held in equilibrium, with the string stretched, by a horizontal force of magnitude kmg.
The line of action of the horizontal force lies in the vertical plane containing the elastic string.

The string AB makes an angle α with the vertical, where $\tan\alpha = \frac{4}{3}$

With the particle in this position, $AB = 5a$, as shown in Figure 1.

(a) Show that $\lambda = \frac{20mg}{3}$

(4)

(b) Find the value of k.

(4)

Question 1 continued

(Total for Question 1 is 8 marks)

2.

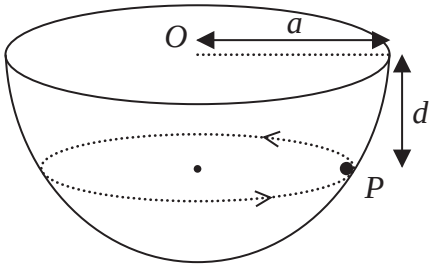

Figure 2

A thin hemispherical shell, with centre O and radius a, is fixed with its open end uppermost and horizontal.

A particle P of mass m moves in a horizontal circle on the smooth inner surface of the shell. The vertical distance of P below the level of O is d, as shown in Figure 2.

(a) Find, in terms of m, g, d and a, the magnitude of the force exerted on P by the inner surface of the hemisphere.

(3)

The particle moves with constant speed v.

(b) Find v in terms of g, a and d.

(5)

Question 2 continued

(Total for Question 2 is 8 marks)

3. A particle P is moving along the x-axis.

At time t seconds, where $t \geqslant 0$, the displacement of P from the origin O is x metres and P is moving with velocity $v\,\text{m s}^{-1}$ in the positive x direction.

The acceleration of P is $\frac{3\sqrt{x+1}}{4}\,\text{m s}^{-2}$ in the positive x direction.

When $t = 0$, $x = 15$ and $v = 8$

(a) Show that $v = (x+1)^{\frac{3}{4}}$

(4)

(b) Find t in terms of v.

(5)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 9 marks)

4. In a harbour, the water level rises and falls with the tides with simple harmonic motion.

On a particular day, the depths of water in the harbour at low and high tide are 4 m and 10 m respectively.

Low tide occurs at 12:00 and high tide occurs at 18:20

(a) Find, in $m\,h^{-1}$, the speed at which the water level is rising on this particular day at 13:35

(6)

A ship can only safely enter the harbour when the depth of water is at least 8.5 m.

(b) Find the earliest time after 12:00 on this particular day at which it is safe for the ship to enter the harbour, giving your answer to the nearest minute.

(4)

Question 4 continued

(Total for Question 4 is 10 marks)

5. A uniform right solid circular cone C has radius r and height $4r$.

(a) Show, using algebraic integration, that the distance of the centre of mass of C from its vertex is $3r$.

[You may assume that the volume of C is $\frac{4}{3}\pi r^3$]

(4)

A uniform solid S, shown below in Figure 3, is formed by **removing** from C a uniform solid right circular cylinder of height r and radius $\frac{1}{2}r$, where the centre of one end of the cylinder coincides with the centre of the plane face of C and the axis of the cylinder coincides with the axis of C.

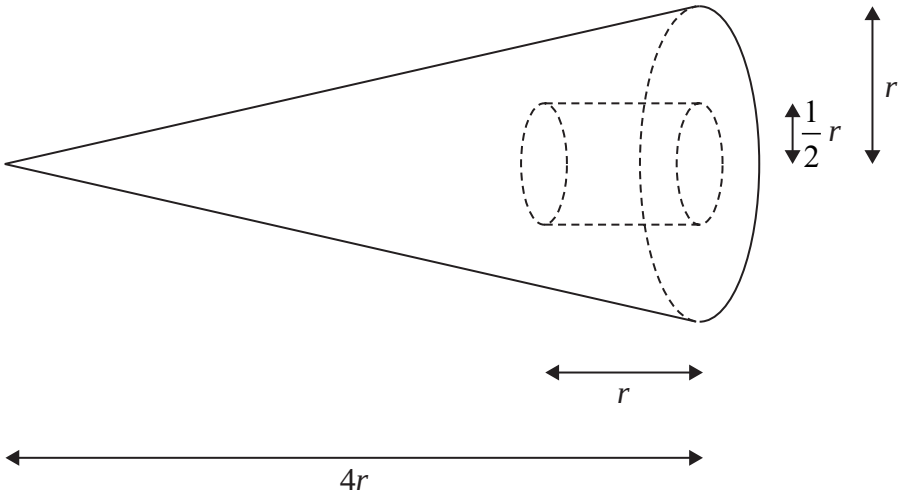

Figure 3

(b) Show that the distance of the centre of mass of S from the vertex of C is $\frac{75}{26}r$

(5)

A rough plane is inclined at an angle α to the horizontal.

The solid S rests in equilibrium with its plane face in contact with the inclined plane.

Given that S is on the point of toppling,

(c) find the exact value of $\tan\alpha$

(3)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 12 marks)

6.

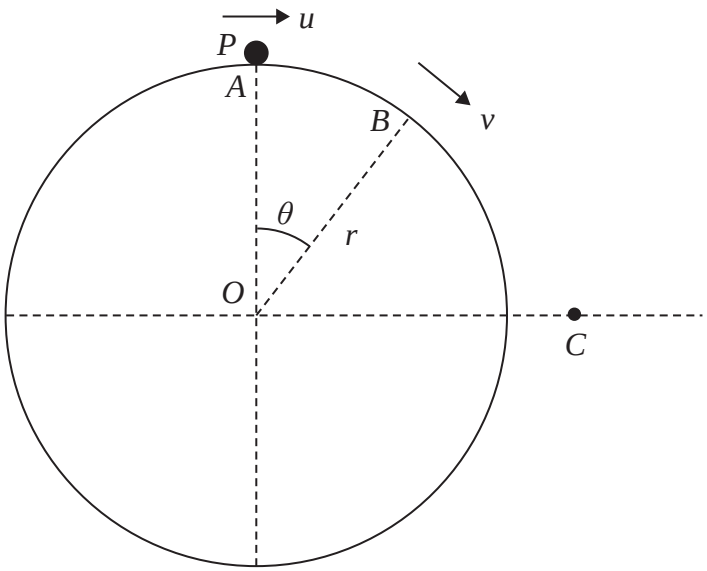

Figure 4

A fixed solid sphere has centre O and radius r.
A particle P of mass m is held at rest on the smooth surface of the sphere at A, the highest point of the sphere.
The particle P is then projected horizontally from A with speed u and moves on the surface of the sphere.
At the instant when P reaches the point B on the sphere, where angle $AOB = \theta$, P is moving with speed v, as shown in Figure 4.

At this instant, P loses contact with the surface of the sphere.

(a) Show that

$$\cos\theta = \frac{2gr + u^2}{3gr}$$

(7)

In the subsequent motion, the particle P crosses the horizontal through O at the point C, also shown in Figure 4.

At the instant P passes through C, P is moving at an angle α to the horizontal.

Given that $u^2 = \frac{2gr}{5}$

(b) find the exact value of $\tan\alpha$.

(6)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 13 marks)

7. A particle P of mass m is attached to one end of a light elastic string of natural length l and modulus of elasticity $2mg$. The other end of the string is attached to a fixed point A on a smooth horizontal table. The particle P is at rest at the point B on the table, where $AB = l$.

At time $t = 0$, P is projected along the table with speed U in the direction AB.

At time t

- the elastic string has not gone slack
- $BP = x$
- the speed of P is v

(a) Show that

$$v^2 = U^2 - \frac{2gx^2}{l}$$

(4)

(b) By differentiating this equation with respect to x, prove that, before the elastic string goes slack, P moves with simple harmonic motion with period $\pi\sqrt{\frac{2l}{g}}$

(5)

Given that $U = \sqrt{\frac{gl}{2}}$

(c) find, in terms of l and g, the exact total time, from the instant it is projected from B, that it takes P to travel a total distance of $\frac{3}{4}l$ along the table.

(6)

Question 7 continued

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS